

Physics

Richardson and Cornell Potentials between Quarks and Antiquarks Inspired by Infrared Asymptotics of the Gluon Propagator

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ABSTRACT. It is shown that the well-known potentials, which give a good description of hadron spectra, can be constructed from infrared asymptotics of the gluon propagator, obtained according to the Dyson-Schwinger equation in gluodynamics. © 2010 Bull. Georg. Natl. Acad. Sci.

Key words: gluon propagator, Dyson-Schwinger equation, infrared asymptotic, light-like gauge.

Potential models are widely used while studying quark-antiquark bound states. Potentials motivated by QCD are based on two kinds of asymptotic: ultraviolet at short distances, i.e. Coulomb-like term and infrared – which in accordance with lattice QCD, the string, Dyson-Schwinger equation etc., gives linear increase of confining potential at large distances. The simple extrapolation of these behaviors is the so-called Cornell potential [1]

$$V(r) = kr - \frac{4}{3} \frac{\alpha_s}{r}, \quad (1)$$

where $\alpha_s = \text{const}$ or is considered as a running coupling interaction [2]

$$\alpha_s(\bar{q}^2) = \frac{48\pi^2}{33 - 2N_f} \frac{1}{\ln(\bar{q}^2 / \Lambda^2)}, \quad \bar{q}^2 \gg \Lambda^2. \quad (2)$$

Here N_f is the number of “effective” fermions and Λ is QCD scale parameter.

Richardson proposed the following potential [2]

$$V(\bar{q}^2) = -\frac{4}{3} \frac{48\pi^2}{33 - 2N_f} \frac{1}{\bar{q}^2 \ln(1 + \bar{q}^2 / \Lambda^2)}, \quad (3)$$

i.e. it is normalized to have correct ultraviolet behavior of α_s for $\bar{q}^2 \gg \Lambda^2$ resulting from perturbation theory, and is extrapolated to infrared region by specific choice of logarithmic function.

Assuming the Richardson potential valid for infrared region as well, after expanding in powers of small $\bar{q}^2$ up to Coulomb term we have

$$V(\bar{q}^2) \approx \frac{64\pi^2}{33 - 2N_f} \frac{\Lambda^2}{(\bar{q}^2)^2} - \frac{32\pi^2}{33 - 2N_f} \frac{1}{\bar{q}^2}, \quad \bar{q}^2 \ll \Lambda^2. \quad (4)$$

Therefore we can model the Cornell potential using this formula.

Further we shall consider pure gluodynamics ($N_f = 0$). In that case we can obtain the values of parameters by comparing Fourier transform of (4) with (1)

$$k^R = \frac{8\pi\Lambda^2}{33}, \quad \alpha_s^R = \frac{2\pi}{11} \approx 0,571. \quad (5)$$

During past years a lot of work has been dedicated

to the investigation of infrared asymptotic of gluodynamics [4-6]. Though this region is non-perturbative, in some cases it is possible to extract not only leading $(\bar{q}^2)^{-2}$, but next to leading Coulomb $(\bar{q}^{-2})$ term as well [7,8]. Therefore it is relevant to ask whether these results are compatible with the potentials mentioned above.

In the paper [9] the Dyson-Schwinger equation for gluon propagator contracted by gauge vector n_μ in the light-cone gauge $(n^2 = 0)$ was studied in the most general case – when both tensor structures of gluon propagator are taken into account. The solution was obtained:

$$\Delta_{\mu\nu}^{ab}(q) = -\frac{i\delta^{ab}}{q^2} \left(-\frac{M^2}{q^2} + b \right) \times \left\{ g_{\mu\nu} - \frac{q_\mu n_\nu + q_\nu n_\mu}{nq} + \lambda \varepsilon q^2 \frac{n_\mu n_\nu}{(nq)^2} \right\} \quad (6)$$

where M^2, b and λ are arbitrary constants and ε is a parameter of the dimensional regularization $(\varepsilon \rightarrow 0^+)$. While obtaining this formula the coefficient of the second (induced) structure was set proportional to ε . It is evident that in the limit $\varepsilon \rightarrow 0$ the contribution of this term in one-gluon exchange potential vanishes. But it gives contribution in loop integrations and therefore the parameter still remains in the right-hand side of Dyson-Schwinger equation. In order to satisfy this equation in the limit $q^2 \rightarrow 0$, we must make use of an additional condition for b , which for $SU(3)_C$ group takes the form [9]

$$1 = \frac{g^2 b}{48\pi^2} (17 + 60\lambda - 9\lambda^2). \quad (7)$$

Let us define a one-gluon exchange potential by means of static $q_0 = 0$ value of propagator (which gives the leading contribution in the quasipotential formalism [10]) and take the color factor into account. Consequently we have

$$V(\bar{q}^2) = -\frac{4}{3} \left[\frac{g^2 M^2}{(\bar{q}^2)^2} + \frac{g^2 b}{\bar{q}^2} \right]. \quad (8)$$

It gives the Cornell potential in coordinate space with the following values of parameters:

$$k = \frac{g^2 M^2}{6}, \quad \alpha_S = \frac{g^2 b}{4\pi}. \quad (9)$$

With regard to (7) $g^2 b$ is expressed by parameter λ , which is arbitrary still and by varying the latter we can obtain any given value of $g^2 b$.

There exists a physically distinguished gauge among the numerous possibilities – the so-called Arbutov gauge [8]. This gauge for $n^2 = 0$ is defined by the following constraint: The leading singular part of gluon propagator in coordinate space must be transversal:

$$x^\mu \tilde{\Delta}_{\mu\nu}^{Sing}(x) = 0. \quad (10)$$

Here $\tilde{\Delta}_{\mu\nu}^{Sing}(x)$ is the Fourier transform of q^{-4} term of propagator (6), which in the arbitrary d -dimensional space has the form [8]:

$$\tilde{\Delta}_{\mu\nu}^{Sing}(x) = \frac{i^{-d} M^2}{16\pi^{d/2}} \times \left\{ \Gamma\left(\frac{d}{2} - 2\right) \left(g_{\mu\nu} \frac{x_\mu n_\nu + x_\nu n_\mu}{(nx)} \right) - \times \left(1 + \lambda \frac{4-d}{2} \right) \Gamma\left(\frac{d}{2} - 3\right) \frac{x^2 n_\mu n_\nu}{(nx)^2} \right\}. \quad (11)$$

Then

$$x^\mu \tilde{\Delta}_{\mu\nu}^{Sing}(x) = -\frac{i^{-d} M^2}{16\pi^{d/2}} (x^2)^{3-d/2} \times \times (2-d/2) \Gamma\left(\frac{d}{2} - 3\right) \frac{n_\nu}{(nx)^2} (1-\lambda). \quad (12)$$

It is clear that the choice $\lambda = 1$ guarantees the transversality of $\tilde{\Delta}_{\mu\nu}$. Choosing this gauge with regard to (7), (9) one obtains

$$\alpha_S = \frac{3\pi}{17} \approx 0.554. \quad (13)$$

This value differs only by 3% from α_S^R (Eq. (5)). If we consider the expression (8) as the first two terms of expansion in Richardson parameterization the corresponding potential in our case will become

$$V(\bar{q}^2) = -\frac{4}{3} \frac{48\pi^2}{34} \frac{1}{\bar{q}^2 \ln(1 + \bar{q}^2 / \Lambda^2)}, \quad (14)$$

where the QCD parameter Λ is determined by the obvious formula:

$$k = \frac{g^2 M^2}{6} = \frac{8\pi\Lambda^2}{34}. \quad (15)$$

The comparison of k and k^R shows that parameter Λ evaluated from the infrared region of gluodynamics via Richardson parameterization is in satisfactory

agreement with its perturbative (i.e. determined by ultraviolet asymptotic) value. Note that if we restrict ourselves only to the first (free) structure ($\lambda = 0$) in gluon propagator the result for α_s and k will change 4 times [7].

To make the picture complete we must incorporate fermions (quarks) as well. Besides it is extremely desirable to investigate this problem in other gauges.

ფიზიკა

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